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Intertemporal Contracts and a Theory of Optimal Auditing

Stefan Krasa
Department of Economics
University of Illinois

Anne P. Villamil
Department of Economics
University of Illinois

Bureau of Economic and Business Research
College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

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Abstract

We study optimal signaling and auditing contracts in an intertemporal economy with asymmetrically informed agents. There is a risk neutral firm and many risk neutral investors. The firm has limited liability, and wishes to raise finance for a project whose returns follow a Markov process. The structure of the Markov process is common knowledge, but only the firm observes the realizations. We derive conditions under which the firm can costlessly signal its state to investors through its choice of payments. For this to occur, returns must be such that the firm can be "rewarded" for truthful reports. When signaling contracts do not exist, we characterize the nature of optimal auditing contracts. We show that if audits occur, they are conducted only in "bad" states. Finally, we derive the Pareto efficient maturity structure of both types of contracts endogenously, show how optimal allocations can be implemented via the appropriate assignment of control rights, and analyze the implications of control for a firm's capital structure.

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Address of the authors: Department of Economics, University of Illinois, 1206 South Sixth Street, Champaign IL 61820

1 Introduction

Firms generally have better information than investors, and revelation of information is essential for investment to occur. This paper focuses on two principal mechanisms commonly used to reveal information: signaling via payment schedules and costly auditing. The following intertemporal investment finance questions motivate our analysis:

(i) Do conditions exist under which a firm can costlessly reveal private information about its state? Can contracts be made contingent on this information? Can the firm and investors write state contingent contracts on non-verifiable private information?

(ii) What is the role of costly auditing in an intertemporal setting? When is it optimal? Should auditing be made contingent on previous firm announcements or should it be imposed exogenously independent of announcements?

(iii) What are the implications of investor versus firm control of productive resources? For example, are optimal investment contracts of fixed maturity or can investors discipline a firm by withdrawing early without incurring self-defeating liquidation penalties?

We analyze these questions in an economy with a firm and risk neutral investors who are differentially informed and wish to write multi-period investment contracts. The investors are endowed with a unit of input (i.e., "wealth") at time zero, are potentially infinitely lived, and consume only once. Their precise time of consumption is uncertain but is determined endogenously. The firm is endowed with a technology whose intertemporal return follows a two-state Markov process, but is penniless (i.e., "wealth constrained"). Investors know the structure of the Markov process but only the firm privately observes the realization each period. A costly auditing technology is available to verify firm reports, and if misreports are detected the firm can be penalized. The penalty is bounded, so the firm has limited liability. We analyze Pareto efficient contracts (relative to the environment) which specify incentive compatible, state contingent payments by the firm to investors. The key problem is to derive contracts which truthfully and efficiently reveal the firm's private information.

Our analysis provides the following answers to the questions posed above. First, we derive computable incentive constraints which, if satisfied, permit the firm and investors to write self-enforcing, state contingent contracts that implement the first best allocation even if the state is not publicly verifiable. These conditions show that the firm must receive a sufficient "reward" in each state for such contracts to exist. Second, when signaling contracts do not exist, we characterize the structure of optimal auditing contracts: If audits occur they are conducted only in bad states. Finally, we derive the Pareto efficient length of both types of contracts endogenously and show that the assignment of control rights is important for implementing optimal contracts.

There is an extensive literature on optimal investment contracts which has addressed (in isolation) some of the interrelated questions posed at the outset of the paper. We will comment on three aspects of this literature. First, the costly state verification model (cf., Townsend (1979) and Gale and Hellwig (1985)) provides a rationale for the use of debt contracts with costly auditing in static environments. These static contracts are optimal (under certain conditions) because they entail fixed payments in non-auditing states and state dependent payments only when auditing occurs, hence they minimize auditing costs. Signaling is never credible in static models because the informed individual always has an incentive to report that the worst state has occurred to minimize payments, and obviously cannot be penalized in the future for misreports made today. More recently, Border and Sobel (1987) study static auditing mechanisms and show that they generally involve "rewards" (i.e., rebates) for truth-telling.

Second, one of the first papers on efficient information revelation in dynamic, stochastic economies with differentially informed agents is Townsend (1982). In a stylized two agent intertemporal model with identically and independently distributed shocks, he shows that multi-period contracts differ substantially from single period contracts. One agent is risk averse and subject to endowment fluctuations and the other agent is risk neutral. The risk neutral agent can sometimes provide consumption insurance to the risk averse agent *without auditing* because he/she can be appropriately compensated in future periods as a function of the risk averse agent's past endowment reports. History dependence is central to this "consumption insurance" approach, and as Phelan and Townsend (1991) note, this has hampered general analytic treatments of the problem. A second class of repeated, incentive constrained models focuses on deriving conditions under which history does not matter and optimal multi-period contracts reduce to a series of single period contracts. Information is revealed (or not revealed) as in static models.

Finally, the problem of who explicitly or implicitly controls the production decision has been raised as an independent but important issue in the contract literature (cf., Hart and Moore (1989) or Aghion and Bolton (1992)). These "control" or "incomplete contracting" models are often contrasted with costly state verification models and have the following structure. First, they assume agents are risk neutral. Second, they assume that contracts cannot be made contingent on some observable states because these states are too difficult to describe or verify in court (this is equivalent to assuming that state verification costs are prohibitively high). Third, they assume that all agents have symmetric information: When a state ω is realized the firm and all investors agree upon it but cannot write contracts contingent on ω because of the second assumption. Fourth, they assume the value of continuation to investors and the firm differs.[1] The main focus of this literature is to study how a priori incomplete initial contracts can be renegotiated in a subsequent period when the commonly agreed upon state is realized.

Our model contains the static costly state verification and incomplete contracting approaches as particular cases of a dynamic, stochastic, differential information contracting model. This permits us to simultaneously analyze questions (i), (ii), and (iii) posed at the outset. This is crucial since we find that the questions are interrelated. In comparison with the existing literature, our paper makes the following contributions. First, we show that intertemporal contracts are important (with and without auditing) because they permit welfare improving information revelation that cannot be achieved in static models. Second, we examine the "control problem" in an economy with differential information where investors and the firm have the ability to write state contingent contracts which can (if necessary) be enforced via costly auditing (i.e., verification costs in the model can vary between zero and prohibitively high levels). In this setting we show: If control is not an important issue then auditing is unnecessary (i.e., when agents can write self-enforcing, incentive compatible signaling contracts); but control is important for implementing auditing contracts.

The paper is organized as follows: Section 2 presents the model. Section 3 considers a benchmark version of the model where both the firm and

[1] Hart and Moore (1989) assume the liquidation value of assets to investors is less than the value to the firm, so investors prefer to renegotiate the loan rather than to "pull the plug" on the firm (because liquidation imposes self-defeating and asymmetric penalties on them). Aghion and Bolton (1992) study the assignment of control and renegotiation when investors and the firm have utility functions which differ with respect to effort.

investors have symmetric information. Section 4 derives conditions for the existence of Pareto efficient signaling contracts and Section 5 analyzes Pareto efficient auditing contracts. Section 6 provides a general contract problem which permits both types of information revelation, analyzes contract implementation via the assignment of control rights, and analyzes firm capital structures. Finally, Section 7 contains concluding remarks.

2 The Model

Consider an economy with m agents where time is indexed by $t = 0, 1, 2, \ldots$. All agents are risk neutral and have a common time preference rate δ. Let one agent be a "firm" who is distinguished by ownership of a production technology but no initial wealth. The remaining agents are "investors" who are distinguished by an endowment of one unit of an investment/consumption good at $t = 0$ (i.e., "wealth"). They can either consume the good in the current time period or invest in the firm. The intertemporal returns from the firm's technology follow a stationary two-state Markov process with $R_t \in \{R_g, R_b\}$ and $R_g > R_b$.[2] The penniless firm wishes to raise finance from the wealthy investors by offering them contracts with payoffs $r(\cdot)$ which may be state and/or history dependent. The main problem is that investors are information constrained: Realization R_t is *privately* observed by the firm at the beginning of period t, and only the structure of the Markov process is common knowledge. Investors have access to an auditing technology which can perfectly reveal R_t and the current value of the firm's assets in any period, but they incur cost c.

To characterize optimal allocations we analyze the problem of an information constrained social planner. The planner writes contracts which specify payments for investors and investment liquidation (i.e., consumption) times for investors and the firm. The firm's information at time t consists of Markov process realizations R_k up to and including time t, denoted by $\mathcal{F}_t$, the σ-algebra generated by R_k, $k = 1, \ldots, t$. Investor and planner information at time t depends on the firm's announced realizations, which need not

[2]Formally, let $\mathcal{S} = \{R_g, R_b\}$ denote the state space, and $p(\rho, \rho')$ denote the transition probability associated with the Markov process, where ρ is the state at time t and ρ' is the state at $t + 1$. Then $(\Omega, \mathcal{F}, P)$ is a probability space which describes the Markov process, where $\Omega = \mathcal{S}^{\mathbb{N}}$ (i.e., the countable product of $\mathcal{S}$).

correspond to the true realizations unless further restrictions are imposed. Denote their information at time t by $\tilde{\mathcal{F}}_t$. They cannot lose information (forget), thus $\tilde{\mathcal{F}}_t \subset \tilde{\mathcal{F}}_{t+1}$.[3] Let τ^1 indicate the period specified by the contract when investors receive payment. Let τ^2 indicate the period specified by the contract when the firm's project is terminated. Clearly, τ^i is an integer valued random variable with $\tau^2 \geq \tau^1$. These investment "stopping times" must have the property that an agent's liquidation time t is contingent solely on the information available to the agent at time t.

Stopping times are defined formally as follows (cf., Ash (1972)):

Definition 1. *Let $\mathcal{F}_t$, $t \in I\!N$, be an increasing sequence of sub σ-fields of $\mathcal{F}$. Given $\mathcal{F}_t$, a stopping time is a map $\tau: \Omega \to I\!N$ such that $\{\tau \leq t\} \in \mathcal{F}_t$ for every $t \geq 0$.*

Stopping times τ are a tool for characterizing history dependent contracts. History dependence is inherent in our model because the firm's termination of investment depends on the entire string of actual realizations $R_1, \ldots, R_t$ and the investors' termination time prescribed by the contract depends on the entire string of announced realizations. These termination rules are stopping times because they cannot depend on future information the respective agents do not yet have.

Assume the Markov Process and information are such that:

(A1) $R_g > R_b > 0$;

(A2) $E\left[\delta^\tau \prod_{k=1}^{\tau} R_k | R_0 = R_b\right] < 1$ for every stopping time $\tau < \infty$ a.e.;

(A3) $\delta E[R_t | R_{t-1} = R_g] > 1$; and

(A4) The initial state is known to be good by all agents.

The interpretation of (A1) is obvious. (A2) indicates that if the economy starts in the bad state, firm finance is not optimal because the expected discounted return is less than the value of consumption, independent of all future investment strategies τ. (A3) indicates that if the state was good last period firm finance is optimal this period. (A4) is obvious and is satisfied by firms that are granted charters or are evaluated by an outside agent (e.g., the government or an underwriter).[4] The fact that such firms are allowed to

[3] We consider truthful information revelation, i.e., $\mathcal{F}_t = \tilde{\mathcal{F}}_t$ for every t, in all contracting problems. Appendix B shows this restriction is without loss of generality.

[4] The initial realization can also be drawn from a known distribution.

"start-up" is a signal that the initial state is good. Useful technical results implied by (A1)–(A4) are proved in Appendix A.

Finally let c, the per-agent cost of state verification, be borne by investors. This cost may be non-pecuniary (e.g., time lost by agents on verification) or pecuniary (e.g., money paid to the auditor). If a firm's misreport is detected by an audit, investors impose a penalty on the firm, denoted by ϕ. Assume the following about the auditing technology and penalty function:

(A5) Monitoring costs satisfy $0 \leq c \leq \infty$.

(A6) Audits reveal R_t and $x_t = \prod_{k=1}^{t} R_k(\omega)$.

(A7) Audit reports are public information.

(A8) The penalty function satisfies $0 \leq \phi(x) \leq x$ for every $x \in I\!R$.

(A5) indicates that contractable symmetric information (i.e., $c = 0$), non-contractable symmetric information (i.e., $c = \infty$), and differential information problems can all be accommodated by the model. (A6) indicates that when auditing occurs, investors learn the current realization (R_g or R_b) and the current value of the firm's assets $x_t = \prod_{k=1}^{t} R_k(\omega)$.[5] (A7) corresponds to many real world auditing situations where audit results are publicly announced. If reports were private, opportunities for delegated monitoring would exist (cf., Diamond (1984) and Krasa and Villamil (1992)). (A8) indicates that the penalties imposed on the firm are bounded by the current value of its assets. Thus, the firm has limited liability. Note that limited liability can ameliorate the effect of (A2) because it limits the firm's losses in the bad state but does not directly affect its profit in the good state.

3 The Symmetric Information Problem

Consider the problem of a social planner who wishes to choose decision rules for the firm and investors at time zero. The planner must find a state contingent interest rate schedule for the firm to offer investors, r, a stopping time for investors, τ^1, and a stopping time for the firm τ^2 which maximize social welfare. To establish a benchmark case, suppose a complete description of the Markov process is common knowledge at time zero *and* the current realization of R_t becomes common knowledge each period. Recall from (A4)

[5] When the Markov process has two states, (A6) implies that audits reveal the exact number of past good and bad Markov process realizations. In the multiple state case, audits are less revealing but the results remain qualitatively similar.

that the initial state is known to be good. The planner solves the following problem at time zero.

Problem 1. Choose $r(\cdot)$, τ^1, and τ^2 to

$$\max_{\tau^1,\tau^2,r} E\left[\delta^{\tau^2}\left(\prod_{k=1}^{\tau^1} R_k - \prod_{k=1}^{\tau^1} r(R_k)\right)\prod_{k=\tau^1+1}^{\tau^2} R_k \,\Big|\, R_0 = R_g\right],$$

subject to:

$$E\left[\delta^{\tau^1}\prod_{k=1}^{\tau^1} r(R_k)\,\Big|\, R_0 = R_g\right] \geq 1. \qquad (IR)$$

The firm's assets at time $\tau^1(\omega)$ are $\prod_{k=1}^{\tau^1(\omega)} R_k(\omega)$. All payments. which are the firm's liabilities, are made at time $\tau^1(\omega)$ when each investor receives $\prod_{k=1}^{\tau^1(\omega)} r(R_k(\omega))$. If $\tau^2(\omega) > \tau^1(\omega)$ then the investment continues and the firm receives a return of $\prod_{k=\tau^1(\omega)+1}^{\tau^2(\omega)} R_k(\omega)$ on its remaining assets. Thus. the objective in Problem 1 is the firm's expected profit. (IR) is an individual rationality constraint which ensures that investors' expected payoff from firm finance is at least unity, the value of consumption.[6]

Using assumptions (A1)-(A4) it is easy to see that in the case of symmetric information the following arrangement is Pareto efficient. Let τ^* denote the optimal stopping time described by the following Lemma.

Lemma 1. *Assume there is symmetric information about the realizations of R_t. Then Pareto efficiency requires investors to continue firm finance as long as the realizations are R_g. The first time a realization R_b is observed investors withdraw their finance and the firm is shut down, i.e., $\tau^1 = \tau^2 = \tau^*$.*

Proof. By (A4) the initial state is g. Thus, (A3) implies it is efficient for investors to refrain from consumption at $t = 0$ since the firm's return exceeds unity. Hence, $\tau^1 > 0$. We first show that $\tau^1 = \tau^*$, i.e., it is optimal to stop investment if R_t switches to R_b. We proceed by way of contradiction. There are two cases: (i) The state is R_b but it is not efficient to withdraw; (i) the state is R_g but it is efficient to withdraw. Case (i) is not optimal by (A2)

[6] Section 6 and Appendix B consider the general case where r is a function measurable with respect to investors' information; r is a function of the current state only here.

because the firm's expected discounted return is less than unity. Case (ii) is excluded by (A3).[7] Thus, investors and the firm can guarantee themselves higher payoffs by continuing in good states and discontinuing investment when R_b is announced. Similarly, $\tau^2 = \tau^*$ since by (A2) it is not optimal for the firm to continue investment in the bad state. This proves the Lemma.

In the remainder of the paper we study two principal contractual arrangements when the firm and investors have *differential information* about R_t: costless signaling by the firm to investors and costly auditing.[8]

4 Informative Interest Rates

Suppose the Markov process realizations R_t are known only by the firm. Is it possible for the firm to reveal its private information truthfully without auditing (while granting investors at least their reservation utility)? For example, consider the extreme case where state verification is impossible because auditing costs are prohibitively high. This assumption is common in the incomplete contracting literature, and can be interpreted as a situation where the state cannot be verified in court. This section shows that even when the state cannot be verified (i.e., $c = \infty$), it is possible to induce the firm to reveal the state truthfully and terminate investment the first time the state switches to R_b as indicated by Lemma 1.[9] The result requires the firm's project to be sufficiently profitable and an intertemporal economy (i.e., no time period in which the economy ends with probability one). We now explain the intuition. At any time t the firm has two options: (a) report truthfully, or (b) misreport. Obviously the firm will choose the option which

[7]Assume that $R_k = R_g$ for $k \leq t$ and that agents withdraw at t. Then the firm's expected profit at t from continuing one more period is $E\left[\delta\left(R_g^t - r(R_g)^t\right) R_{t+1} \mid R_t = R_g\right]$. If investors stay with the firm for one more period then (IR) still holds and the firm's profit is given by $E\left[\delta\left(R_g^t R_{t+1} - r(R_g)^t r(R_k)^{t+1}\right) \mid R_t = R_g\right]$. Since $r(R_k) \leq R_k$, for every k the firm's profit is higher if investors do not withdraw.

[8]"Brute force" information revelation by bond posting is not possible because the firm is wealth constrained. Implicit bond posting, by liquidating assets at time t, showing them to investors to prove the state is good, and reinvesting them at $t+1$, is also not optimal by Lemma 1 since it lowers the base on which the firm earns its return.

[9]This τ^* result also holds in the general case where auditing costs are not prohibitively high (i.e., $0 \leq c < \infty$).

maximizes its expected profit. To approximate expected profit in a simple case, restrict attention to the firm's net-worth at $t+1$. Further, assume that the firm announces R_b at $t+1$ independent of the actual realization.

First, suppose the economy is in state R_g at time t. Will the firm report the good state truthfully? In case (a) the firm reports R_g truthfully and its net-worth at $t+1$ is $R_g^{t+1} - r_g^t r_b$ if $R_{t+1} = R_g$ and $R_g^t R_b - r_g^t r_b$ if $R_{t+1} = R_b$. In case (b) the firm reports R_b, by Lemma 1 investors withdraw their finance, and the firm is left with $\tilde{x}_t = R_g^t - r_g^{t-1} r_b$ at time t. Because the true state is good, the firm continues to invest at time $t+1$ with residual assets $\tilde{x}_t$, thus its net-worth at $t+1$ in case (b) is $\tilde{x}_t R_g$ if $R_{t+1} = R_g$ and $\tilde{x}_t R_b$ if $R_{t+1} = R_b$. Simple algebra reveals that there is a gain of $r_g^{t-1} r_b (R_g - r_g) > 0$ from truthful reporting if $R_{t+1} = R_g$, and a loss of $r_g^{t-1} r_b (R_b - r_g) < 0$ from truthful reporting if $R_{t+1} = R_b$.[10] Thus, there are two effects which determine whether the firm will report R_g truthfully, but they work in opposite directions: First, the firm loses by falsely reporting R_b because its assets are reduced from x_t to $\tilde{x}_t$ due to withdrawals and profit is an increasing function of total assets. Second, it may gain from misreporting because it eliminates its future liabilities. If r_g and the probability of switching from the good to the bad state ($p(g,b)$) are sufficiently small (i.e., the project is sufficiently profitable) the first effect dominates the second and the firm will not misreport in the good state. The intertemporal structure of the model is important for this incentive constraint to hold. For example, assume that the economy ends at time T. In the absence of auditing it is always optimal for the firm to report the bad state at T independent of the actual realization. This is not the case in the intertemporal model since an announcement of R_b induces withdrawal of firm finance which in turn reduces the firm's profit in future time periods.

Now suppose the economy is in state R_b at time t. Will the firm report the bad state truthfully? If the firm falsely reports R_g and continues, its expected return is lowered by (A2). Because limited liability ameliorates the impact of (A2) in low net-worth states the contract must guarantee the firm sufficiently high returns in both the good and bad states so it can build up enough net-worth over time to ensure it has something to lose from lying. This incentive constraint is not necessary in the last period of a finite (or one period) model. For example, if the economy ends in some period T

[10] The sign of the two inequalities follows since $r_g \leq R_g$, $r_b \leq R_b$ and $r_g > R_b$. The inequality $r_g > R_b$ is implied by individual rationality.

and $R_T = R_b$, the firm will always report the state truthfully since $r_b < r_g$. However, in an intertemporal economy where the firm's net-worth in time period T is sufficiently low and the firm is "protected" by limited liability, the firm may have an incentive to misreport.

In conclusion, when the returns in each state are sufficient it is in the firm's interest to report truthfully because lying reduces its expected profit. The intertemporal structure of the model is essential for ensuring that the firm reports truthfully in the good state, but it (in conjunction with limited liability) also gives the firm an incentive to misreport and continue in the bad state. In the remainder of this Section we derive formally two incentive constraints which ensure truthful reporting:

(ICCb) ensures the firm truthfully reports the bad state when it occurs; and

(ICCg) ensures the firm truthfully reports the good state when it occurs.

When these constraints are satisfied, investment is terminated according to Lemma 1 (i.e., the first time R_b is realized).

Consider the following information constrained problem at time zero.

Problem 2. Choose r_g, r_b to

$$\max_{r_b, r_g} E\left[\delta^{\tau^*}\left(R_g^{\tau^*-1}R_b - r_g^{\tau^*-1}r_b\right) \,\middle|\, R_0 = R_g\right];$$

subject to:

$$E\left[\delta^{\tau^*} r_g^{\tau^*-1} r_b \,\middle|\, R_0 = R_g\right] \geq 1; \tag{IR}$$

$$R_g^{t-1}R_b - r_g^{t-1}r_b \geq E\left[\delta^{\tau}\left(R_g^{t-1}R_b \prod_{k=t+1}^{t+\tau} R_k - r_g^{t+\tau-1}r_b\right)^+ \,\middle|\, R_t = R_b\right]$$

for every stopping time τ; $(ICCb)$

$$r_g \leq \frac{R_b}{1 - \delta p(g,g)(R_g - R_b)}. \tag{$ICCg$}$$

Problem 2 is identical to Problem 1 except that incentive constraints (ICCb) and (ICCg) are included and it incorporates the Pareto efficient stopping time τ^*. We now derive (ICCb) and (ICCg).

Derivation of (ICCb). By (A4) the state is publicly known to be good at the outset. Assume it switches to b at time t. If the firm truthfully announces b it must pay investors $r_g^{t-1}r_b$, and its profit in this case is given by

$$\delta^t \left(R_g^{t-1}R_b - r_g^{t-1}r_b\right). \tag{1}$$

If the firm misreports (i.e., announces g so investors continue to invest) it must pay them r_g^{t+s} at time $t+s$. In general the time when the firm induces investors to withdraw is a stopping time τ. Given τ, the firm's profit from misreporting is

$$E\left[\delta^{t+\tau}\left(R_g^{t-1}R_b\prod_{k=t+1}^{t+\tau}R_k - r_g^{t+\tau-1}r_b\right)^+ \middle| R_t = R_b\right]. \tag{2}$$

(2) is derived as follows: If the firm is able to honor its payment to investors (i.e., $R_g^{t-1}R_b\prod_{k=t+1}^{t+\tau}R_k \geq r_g^{t+\tau-1}r_b$), investors will not know that misreports occurred, but they will know that misreports occurred if the firm is insolvent (i.e., $R_g^{t-1}R_b\prod_{k=t+1}^{t+\tau}R_k < r_g^{t+\tau-1}r_b$). These "off-equilibrium path" misreports amount to fraud (rather than simple bankruptcy states as in static costly state verification models), and the firm is penalized by (A8). Hence, the firm's payoff from fraud is characterized by the positive part of the difference between its assets and liabilities.[11] (1) and (2) immediately yield (ICCb).

(ICCb) is simplified in Lemma 2 because it has a useful economic interpretation explained after the proof.

Lemma 2. *(ICCb) is equivalent to the following constraint:*

$$r_b \leq a(\tau)R_b, \textit{ for every stopping time } \tau, \tag{3}$$

where $a(\tau)$ is the largest a which fulfills

$$E\left[\delta^\tau \frac{\left(\prod_{k=1}^{\tau}R_k - ar_g^\tau\right)^+}{1-a} \middle| R_0 = R_b\right] \leq 1. \tag{4}$$

Proof. Define $\gamma_i = r_i/R_i$, for $i = g, b$. Then (ICCb) becomes

$$\left(1-\gamma_g^{t-1}\gamma_b\right)R_g^{t-1}R_b \geq E\left[\delta^\tau\left(R_g^{t-1}R_b\prod_{k=t+1}^{t+\tau}R_k - r_g^{t+\tau-1}r_b\right)^+ \middle| R_t = R_b\right],$$

[11] We show it is optimal to set $\phi(x) = x$ in Appendix B.

which is (4) when $a = \gamma_g^{t-1}\gamma_b$.[12] On $(-\infty, 1]$, observe that the function $a \mapsto \left(\prod_{k=1}^{\tau(\omega)} R_k(\omega) - a r_g^{\tau(\omega)}\right)^+ / 1 - a$ is increasing if $\prod_{k=1}^{\tau(\omega)} R_k(\omega) \geq r_g^{\tau(\omega)}$ and decreasing otherwise.[13] Claim 1 in Appendix A shows the function $\psi\colon [0,1) \to I\!R$ defined by the left-hand-side of (4) is U-shaped and thus assumes only one local minimum. We now use this fact to prove the Lemma. Fix τ in (4). Since ψ is U-shaped, there exist a_1, a_2 with $0 \leq a_1 \leq a_2 \leq 1$ such that (4) holds for all a with $a_1 \leq a \leq a_2$ and it is violated otherwise. From this it follows that $a_1 = 0$. Assume by way of contradiction that $a_1 > 0$. For $a = 0$, (4) reduces to (A2) and is therefore fulfilled. However, this is a contradiction to the assumption that (4) is violated for all $a \in [0, a_1)$. Thus, for every τ there exists an $a(\tau) \leq 1$ such that (ICCb) holds for $\gamma_g^{t-1}\gamma_b \leq a(\tau)$. This must hold for every $t = 1, 2, \ldots$ and since $\gamma \leq 1$, this is equivalent to $\gamma_b \leq a(\tau)$. Thus, (ICCb) is equivalent to (3) and (4). This proves the Lemma.

Remark 1. Lemma 2 shows that (ICCb) bounds the percentage of payoffs the firm can offer investors in the bad state. When this bound is satisfied, the firm reveals truthfully the bad state when it occurs. (3) indicates there must be some "spread" between R_b and r_b for optimal contracts which truthfully signal R_b to exist, i.e., $a(\tau)$ is less than one for every τ. This follows from (4) when $r_b < R_b$. Remark 2 will show that $r_g < R_g$, which means there must also be sufficient "spread" in the good state for optimal signaling contracts to exist. The intuitive idea is that the difference between what a firm receives (R_t) and what it must pay (r_t) must be sufficiently large to "reward" the firm for telling the truth. This finding in our intertemporal model is reminiscent of Border and Sobel's (1987) finding in a static context that firms must be rewarded to tell the truth and that such rewards lower expected auditing costs. In their static model at least some auditing remained necessary, but in our intertemporal model we show that rewards can sometimes be paid such that no auditing is necessary (cf., the Example below). In an intertemporal model the mere threat of seizing the firm's assets can reduce or even eliminate the ex post inefficiency of auditing that occurs in static models.

Derivation of (ICCg). Assume the state is g at time t, and that the firm reveals the state truthfully at t and in future periods. Suppose the state

[12] Divide both sides by $\left(1 - \gamma_g^{t-1}\gamma_b\right) R_g^{t-1} R_b$, change the summation, and condition with respect to R_0 instead of R_t. This can be done because the Markov process is stationary.

[13] Clearly, the function is constant if $\prod_{k=1}^{\tau(\omega)} R_k(\omega) = r_g^{\tau(\omega)}$.

switches to b at time $t+k$. The investors' payment at time $t+k$ is $r_g^{t+k-1} r_b$ and the firm's return is $R_g^{t+k-1} R_b$. Thus, the firm's expected profit is

$$\sum_{k=1}^{\infty} p(g,g)^{k-1} p(g,b) \delta^{t+k} \left(R_g^{t+k-1} R_b - r_g^{t+k-1} r_b \right). \tag{5}$$

Now consider the alternative case where the firm misreports, announcing b at time t. This causes investors to withdraw, receiving $r_g^{t-1} r_b$. Since the state is g it is optimal for the firm to induce continued investment until the state switches to b. The firm's expected profit in this case is given by[14]

$$\sum_{k=1}^{\infty} p(g,g)^{k-1} p(g,b) \delta^{t+k} \left(\left[R_g^t - r_g^{t-1} r_b \right] R_g^{k-1} R_b \right). \tag{6}$$

Without loss of generality assume feasibility holds under truthful reporting, so $R_g^{t+k-1} R_b - r_g^{t+k-1} r_b > 0$. Thus, (5) and (6) imply that (ICCg) holds if

$$\sum_{k=1}^{\infty} \delta^{k-1} p(g,g)^{k-1} p(g,b) r_g^k \leq \sum_{k=1}^{\infty} \delta^{k-1} p(g,g)^{k-1} p(g,b) R_g^{k-1} R_b. \tag{7}$$

Using the formula for the geometric series, (7) immediately simplifies to $r_g/(1-\delta p(g,g) r_g) \leq R_b/(1-\delta p(g,g) R_g)$, which is equivalent to (ICCg).

Remark 2. (ICCg) can be written in the form $r_g \leq k R_g$, where $k = R_b/R_g[1-\delta p(g,g)(R_g - R_b)] < 1$. Note that (ICCg) restricts r_g, while (ICCb) depends on r_g and r_b (i.e., sufficient profitability in both states is required). Very "high" values of r_b and r_g violate the constraints.[15] Thus, as in Border and Sobel the firm must be rewarded to make it incentive compatible for it to announce the true state. This is not possible if R_t is only marginally higher than r_t. If the "spread" is not sufficiently large, optimal signaling contracts will not exist.

A formal proof of existence of perfect signaling contracts is given in Krasa and Villamil (1993). We now construct a simple example to show how such contracts are derived. We begin by checking that conditions (A1)–(A3) hold

[14] $p(g,g)^{k-1} p(g,b)$ is the probability of a string of $k-1$ good states and one bad state and $\delta^{t+k} \left(R_g^{t+k-1} R_b - r_g^{t+k-1} r_b \right)$ is the discounted return between time t and time $t+k$.

[15] In the extreme case where $r_g = R_g$, (ICCg) implies $R_g \leq R_b$, a contradiction to (A1).

(using Lemma A in Appendix A) and assume that (A4)–(A8) hold directly. Next use (ICCg) to compute the upper bound for r_g, thus finding all r_g which fulfill (ICCg). From Lemma 2 it follows that $a(\tau)$ decreases as r_g decreases (i.e., (ICCb) is more slack the higher r_g). Next choose r_g such that (ICCg) holds with equality. To fulfill (IR), r_b must be chosen such that $p(g,g)r_g + p(g,b)r_b = 1$. Thus, we must show that (4) holds for $a = r_b/R_b$. To compute (4), use Corollary 2 in Appendix A, which shows that $\max_{\tau\in\mathcal{H}} E\left[\prod_{k=1}^{\tau} R_k \,\middle|\, R_0 = R_b\right] = \delta p(b,g)\tilde{R} + \delta p(b,b)R_b$, where $\tilde{R} = R_g + \delta p(g,b)R_b/(1-\delta p(g,g)R_g)$. Hence, Lemma 2 implies that (ICCb) holds if

$$\frac{1}{1-a}\delta\left(p(b,g)R_g + \frac{\delta p(b,g)p(g,b)R_b}{1-\delta p(g,g)R_g} + p(b,b)R_b\right) < 1. \qquad (8)$$

The following numerical example illustrates this procedure:

Example. Assume $R_g = 1.15$, $R_b = 0.7$, $p(g,g) = 0.86$, $p(b,b) = 0.99$, so $p(g,b) = 0.14$ and $p(b,g) = 0.01$. (A1) and (A3) obviously hold; check that (10) from Lemma A in Appendix A is fulfilled. Thus, by Lemma A condition (A2) is fulfilled. Now check that (ICCg) holds with equality for $r_g = 1.1420$. (IR) requires $p(g,g)1.1420 + p(g,b)r_b = 1$, and hence $r_b = 0.1282$, so $a = r_b/R_b = 0.1831$. Thus, the left-hand side of (8) is 0.9715, the condition is fulfilled, and (ICCb) holds, so this is a perfect signaling contract.

5 Information and Auditing

Again suppose the Markov process realizations are known only by the firm. We now consider the social planner's problem when signaling contracts do not exist and costly auditing is used to reveal information. We prove three technical results that are essential for an analysis of control and auditing in Section 6. Lemma 3 allows us to restrict attention to misreports by the firm that cannot be detected by auditing. Lemma 4 proves that optimal stopping time τ^* from Lemma 1 remains optimal in the auditing model. However, the equilibrium is no longer first best. Finally, the Theorem proves that if audits occur they are conducted only in the bad state. Recall from Section 2 that investors can verify Markov process realizations R_t if they bear a cost c and that audits publicly reveal information.[16] Although audits reveal the state

[16] Audits reveal the firm's current realization, R_g or R_b, and asset value $x = \prod_{k=1}^{t} R_k(\omega)$.

without error, the firm still has a propensity to misreport because of limited liability (recall that limited liability weakens (A2)).

We begin by modeling firm reports and the investors' auditing decision. For every $k \in I\!N$, let h_k be a report function which indicates the Markov process realization reported by the firm to investors at time k, where ω denotes the true state and $h_k: \Omega \rightarrow \mathcal{S} = \{R_g, R_b\}$ is $\mathcal{F}_k$-measurable. Further, let $h(\omega) = (h_1(\omega), h_2(\omega), \ldots)$.[17] For $i \in I\!N$, let ς^i be a stopping time which denotes the ith time investors audit.[18] The information revealed by audits is reflected in the investors' information sets. In this Section we restrict attention to full information revelation, so no additional information is revealed from auditing. In equilibrium audits are purely a disciplinary device which deter misreports, and there is ex post contract enforcement. Appendix B considers the general case where audits need not be fully revealing. Assume that audits do not occur after investors have withdrawn their finance (i.e., if $\varsigma^i(\omega) > \tau^1(\omega)$). A truth telling contract is a tuple $(\tau^1, \tau^2, (r_k)_{k \in I\!N}, (\varsigma_k)_{k \in I\!N})$ which maximizes the firm's profit subject to an individual rationality constraint for investors and an incentive constraint. The incentive constraint specifies that the firm cannot increase its profit by misreporting the state.[19]

Appendix B proves the following result.

Lemma 3. *h can be restricted to misreporting strategies which cannot be detected via auditing. Formally, h can be restricted to the set $\mathcal{H}(\varsigma^i, \tau^1) = \{h$: If $\varsigma_i(h(\omega)) \le \tau^1(h(\omega))$ for some i then $h_k(\omega) = R_k(\omega)$ for every $k \le \varsigma_i(h(\omega))\}$.*

If the firm's reports follow h, and misreports are not detected via auditing, the investors' information set at time t is $\tilde{\mathcal{F}}_t$, the σ-algebra generated by $h_1, \ldots, h_k$. Investors' decisions regarding when to audit and when to stop investing are functions of the announced state, which need not be the true state without additional structure. In particular, the investors' stopping rule is given by $\tau^1(h)$ and the auditing rules by $\varsigma^i(h)$, $i \in I\!N$. In Appendix B we prove via a revelation principle argument that we can restrict our analysis

[17] $h: \Omega \rightarrow \Omega$ when Ω is identified with $\mathcal{S}^{I\!N}$, the set of sequences $(R_k)_{k \in I\!N}$ where $R_k = \{R_g, R_b\}$.

[18] ς^i is defined with respect to investors' information $\tilde{\mathcal{F}}_t$, $t \in I\!N$ such that $\varsigma^i(\omega) \le \varsigma^{i+1}(\omega)$ a.e. for every $i \in I\!N$. $\varsigma^i \equiv \infty$ yields the case without auditing.

[19] Misreporting is the choice of a function h with $h(\omega) \ne \omega$ on a set of positive measure.

without loss of generality to truth-telling contracts. Thus, the following Pareto problem results:

Problem 3. Choose $(\tau^1, \tau^2, (r_k)_{k\in\boldsymbol{N}}, (\varsigma_i)_{i\in\boldsymbol{N}})$ to

$$\max E\left[\delta^{\tau^2(\omega)}\left(\prod_{k=1}^{\tau^1(\omega)} R_k(\omega) - \prod_{k=1}^{\tau^1(\omega)} r_k(\omega)\right)\prod_{k=\tau^1(\omega)+1}^{\tau^2(\omega)} R_k(\omega) \;\Big|\; R_0 = R_g\right]$$

subject to:

$$E\left[\delta^{\tau^1(\omega)}\prod_{k=1}^{\tau^1(\omega)} r_k(\omega) \;\Big|\; R_0 = R_g\right] - E\left[\sum_{k=1}^{\tau^1(\omega)} cn_k(\omega) \;\Big|\; R_0 = R_g\right] \geq 1; \qquad (IR)$$

$$E\left[\delta^{\tau^2(\omega)}\left(\prod_{k=1}^{\tau^1(\omega)} R_k(\omega) - \prod_{k=1}^{\tau^1(\omega)} r_k(\omega)\right)\prod_{k=\tau^1(\omega)+1}^{\tau^2(\omega)} R_k(\omega) \;\Big|\; R_0 = R_g\right]$$
$$\geq E\left[\delta^{\tilde{\tau}^2(\omega)}\left(\prod_{k=1}^{\tau^1(h(\omega))} R_k(\omega) - \prod_{k=1}^{\tau^1(h(\omega))} r_k(h(\omega))\right)^+ \prod_{k=\tau^1(h(\omega))+1}^{\tilde{\tau}^2} R_k \;\Big|\; R_0 = R_g\right],$$

for all $h \in \mathcal{H}(\varsigma_i, \tau^1)$, and for $\tilde{\tau}^2$ with $\tilde{\tau}^2(\omega) \geq \tau^1(h(\omega))$ a.e. $\qquad (ICC)$

This Pareto problem with auditing is solved by the planner at time zero. The objective is the firm's expected profit and (IR) is the investors' individual rationality constraint. (ICC) is an incentive constraint which restricts the firm to misreports not ruled out by Lemma 3, where the left-hand side is the firm's payoff from truthful reports and the right-hand side is its payoff from misreports h (and the firm replaces τ^2 by $\tilde{\tau}^2$).[20]

We now show that the symmetric information stopping times remain optimal.

Lemma 4. *If c is sufficiently small, $\tau^1 = \tau^2 = \tau^*$ for any optimal contract.*

[20]After investors withdraw, the firm can choose an arbitrary investment policy. Thus, (ICC) must hold for every $\tilde{\tau}^2 \geq \tau^1(h)$, where h and $\tilde{\tau}^2$ describe all possible state-contingent paths for the firms' misreporting strategies.

Proof. We proceed by way of contradiction. There are two possible cases: (a) a state exists with an R_b realization where investors and the firm do not cease investment; or (b) a state exists with an R_g realization where investors cease investment. Consider (a), and let t be the first time that (a) occurs. The firm's value at t is given by $R_g^{t-1}R_b$. From Lemma A in Appendix A it follows that the loss of expected utility from continuing investment in the bad state is $R_g^{t-1}R_b l$, where

$$l = 1 - \delta\left(p(b,b)R_b + p(b,g)R_g + \delta p(b,g)p(g,b)R_b/(1-\delta p(g,g)R_g)\right) > 0.$$

This loss remains bounded away from zero for every t, since $R_g > 1$.[21] Now consider (b), and assume that until time t the realizations of R_k are R_g. The firm's value at t is given by R_g^t. (A3) implies that the loss in terms of expected utility from not continuing investment in the good state is given by $R_g^t l$, where $l = E[R_k \mid R_{k-1} = R_g] - 1 > 0$. This term is bounded away from zero for every t. Finally, consider a contract where $\tau_2 \neq \tau^*$. Similar arguments show there is an expected utility loss which remains bounded away from zero. Thus, if auditing costs are sufficiently small optimality requires $\tau^1 = \tau^*$ and $\tau^2 = \tau^*$.

Lemmata 3 and 4 prove we can assume without loss of generality that all contracts involve no detectable misreports and a choice of $\tau^1 = \tau^2 = \tau^*$, respectively. We now derive the set of information constrained Pareto efficient auditing rules.

Theorem. *Assume an optimal auditing contract exists and that $R_k = R_g$ for every $k < t$. At time t the auditing scheme must specify that either auditing never occurs or auditing occurs only if R_b is announced.*

Proof. From Lemma 4 it follows that the optimal contract involves termination of investment if R_b is announced. Thus, the set of all possible auditing rules in each time period is given as follows: (a) never audit, (b) audit only if the good state is announced, (c) audit only if the bad state is announced, or (d) audit independent of the announced state. To prove the Theorem it is sufficient to show that only (a) and (c) can occur. Thus, the remainder of the proof shows that (c) dominates (d) and (d) dominates (b). Hence (c) also dominates (b).

[21] $R_g < 1$ implies $R_b < 1$ by (A1) and hence this violates (A3).

We first show that (d) dominates (b). Assume a contract requires auditing at time t if the good state is announced. Replace this contract by uncontingent verification at $t+1$ and no verification at t. Expected state verification costs remain the same if previously there was no auditing at $t+1$, and they strictly decrease otherwise.[22] Further, note that all misreports which are detected under the original contract are also detected under the alternative contract. Thus, any contract which specifies auditing in the good state for some time periods can be replaced by a contract where auditing never takes place contingent on R_g being announced. Thus, (d) dominates (b).

We next prove that (c) dominates (d). Assume a contract requires uncontingent verification at time t. Replace this contract with one which specifies verification at t if R_b is announced and uncontingent verification at $t+1$. Expected auditing costs are the same if the original contract implied no verification at $t+1$ and they strictly decrease otherwise. All misreports which are detected under the original contract are detected under the alternative contract. Repeating this argument, replace contracts which specify auditing of type (d) in some time periods by contracts which imply in any given time period either no auditing or auditing contingent on R_b being announced. Thus, expected auditing costs decrease. This concludes the proof.

Remark 3. The characterization of the optimal auditing structure given by the Theorem is an intertemporal version of Townsend's (1979) static lower interval result. If auditing occurs, it occurs only in the bad state. In contrast to the static costly state verification model, our intertemporal model provides insight into the use of debt versus equity.[23] We consider this capital structure problem in the next Section.

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[22]If the original contract did not specify auditing at $t+1$, auditing occurs under the original contract whenever it occurs under the alternative contract. This is the case since if the good state is announced at t auditing occurs (so no misreporting is possible), and it is optimal to continue to period $t+1$. Alternatively if the bad state occurs, investment is stopped and $t+1$ is never reached. If the original contract specified auditing at $t+1$ expected auditing costs obviously decrease.

[23]The static costly state verification model explains the use of debt, but is unable to explain the use of equity.

6 Control and Contract Implementation

In Sections 4 and 5 we characterized Pareto efficient allocations for the firm and investors. We now ask the following questions: Is it possible to implement the optimal contract with Pareto efficient stopping time τ^* by making the investment termination decision endogenous (i.e., by giving one or both parties the right to terminate investment)? Further, does the implementation mechanism provide insight into the firm's choice of capital structure? The first question is important because contracts are not routinely written with pre-specified investment termination times. Rather, contracts typically specify conditions under which one or both parties have the right to decide the firm's future. The second question amounts to asking why firms choose debt versus equity (or both), a vexing problem in finance. We answer these questions by showing that contracts which assign control rights "appropriately" implement the constrained Pareto efficient state contingent investment plan.[24] We focus on the two most commonly observed types of control contracts: equity (voting and non-voting) and debt.[25] As in Aghion and Bolton (1992), we regard the distinguishing feature between debt and equity to be the assignment of property rights (not contract payoffs).

The Section proceeds as follows: First, the results from Section 5 are used to simplify Problem 3. The resulting Problem 4 is general enough to include Problem 2 as special case.[26] Next, constraints on the firm and investors are used to study implementation of the Pareto efficient allocation characterized by Problem 4 via debt and equity contracts. We show that by assigning control rights "appropriately" the solution to Problem 4 can be implemented, and hence optimal investment plan τ^* can be achieved. We then classify conditions under which each type of contract is optimal, and find that the appropriate control structure depends on the primitives of the

[24] Methodologically our approach is similar to that in Gale (1991), i.e., we show that in many cases relatively simple contracts can implement the efficient outcome.

[25] Voting equity is any contract which assigns all control rights to investors, so they decide whether or not the firm should continue. Non-voting equity assigns control rights exclusively to the firm, so it decides whether or not to continue. Debt assigns contingent control rights: in good states the firm retains ownership, but in bad (bankruptcy) states ownership is transferred to debt holders. Thus, the firm has control in good states but investors have control in bad states.

[26] In addition, it may be non-recursive because auditing is contingent on the time period. Stopping times are crucial for characterizing analytic solutions for such problems.

economy (i.e., which constraints bind). Finally, the implications of these results for a firm's capital structure are discussed.

Consider first some notation. Given a history dependent payment schedule $(r_k)_{k\in I\!N}$, let $D_t = \prod_{k=1}^{t} r_k(\omega)$, where ω is any state for which $R_k(\omega) = R_g$ for every $k = 1, \ldots, t-1$ and $R_t(\omega) = R_b$. Thus, D_t are the firm's liabilities at time t after $t-1$ good realizations and one bad realization. Let $A \subset I\!N$ denote the set of all time periods where audits occur if R_b is announced. Problem 3 can now be written in the following form.

Problem 4. Choose $((D_k)_{k\in I\!N}, A)$ to

$$\max \sum_{t=1}^{\infty} p(g,g)^{t-1} p(g,b)\delta^t \left(R_g^{t-1}R_b - D_t\right)$$

subject to:

$$\sum_{t=1}^{\infty} p(g,g)^{t-1}p(g,b)\delta^t D_t - \sum_{t\in A} cp(g,g)^{t-1}p(g,b) \geq 1 \qquad (IR)$$

$$R_g^{t-1}R_b - D_t \geq E\left[\delta^{\tau}\left(R_g^{t-1}R_b \prod_{k=t+1}^{\tau} R_k - D_{t+\tau}\right)^{+} \;\middle|\; R_t = R_b\right]$$

for every t in $I\!N$ and every stopping time τ such that $\tau(\omega) \notin A$. $\qquad (ICb)$

$$\sum_{s=1}^{\infty} p(g,g)^{s-1}p(g,b)\delta^{t+s}\left(R_g^{t+s-1}R_b - D_{t+s}\right)$$
$$\geq \sum_{s=1}^{\infty} p(g,g)^{s-1}p(g,b)\delta^{t+s}\left(R_g^{t} - D_t\right)R_g^{s-1}R_b,$$

for every $t \notin A$. $\qquad (ICg)$

The objective and (IR) in Problem 4 are identical to those in Problem 3 (use the fact that $\tau^1 = \tau^2 = \tau^*$, replace the expectations with probabilities, and use the D_t notation), but as in Problem 2 there are two "state-wise" incentive constraints. (ICb) ensures it is optimal to announce R_b if it actually occurs (where τ in (ICb) corresponds to an undetectable misreporting strategy h which reports R_b only in time periods $t \notin A$)[27] and (ICg) ensures it is optimal

[27] h can be completely described by τ since investors stop if R_b is announced. Use of the stopping time τ instead of misreporting function h facilitates analysis of the problem.

to announce R_g if it occurs. As in Problems 1, 2, and 3, (IR) is the individual rationality constraint at time zero.

To study implementation of the solution to Problem 4 we will analyze two sets of constraints. First, consider the firm's incentive constraints from Problem 4. Let (D_t, A) be a solution to Problem 4. Incentive constraints (ICb) and (ICg) describe four possible cases for $(D_t, \emptyset)$:[28] (ICb) and (ICg) both hold; only (ICg) holds; only (ICb) holds; and neither holds. Second, consider the constraints which affect investors. The intertemporal version of the investors' individual rationality constraint from Problem 4 is:

$$D_t \leq \sum_{k=1}^{\infty} p(g,g)^{k-1} p(g,b) D_{t+k}, \text{ for every } t \in I\!N. \qquad (IIRg)$$

(IIRg) ensures that investors continue firm finance when the firm announces R_g, in accordance with optimal investment plan τ^*. In (IIRg) as well as in constraint (IICb) below we assume that "off-equilibrium path" payoffs to investors are also D_t. For example, if investors decide to continue firm finance at some time k despite the fact that the state is bad, they receive a payment D_t at time $t > k$ if finance is terminated at time t. This assumption simplifies the analysis and notation. Note that these off-equilibrium path payoffs to investors are not relevant in Pareto Problem 3 since investors cannot renege on the contract and continue investing in the bad state.

The following condition ensures that investors terminate finance when the firm announces R_b:

$$E\left(\min\left\{ \delta^\tau D_\tau, \prod_{k=1}^{t+\tau} R_k \right\} \,\middle|\, R_t = R_b \right) \leq D_t, \qquad (IICb)$$

for every t and for every stopping time τ.[29] Thus, (IICb) and (IIRg) also describe four possible cases: (IICb) and (IIRg) both hold, only (IICb) holds, only (IIRg) holds, and neither holds. Finally, (A2) implies that at any time t, (IICb) and (ICb) cannot both be violated for the contract $(D_t, \emptyset)$. That is, when auditing does not occur investors and the firm cannot both wish to

[28]Note that we replace (D_t, A) by a contract with the same payment schedule but no auditing.

[29]The left-hand side of (IICb) is the continuation payoff, which is the minimum of the two payoffs since the firm cannot make payments in excess of the value of its assets. D_t is the investors' payoff if investment stops at time t.

continue in the bad state because (A2) implies that the payoff from doing so is too low.

We now discuss how particular types of commonly observed control contracts can be used to implement the optimal solution to Problem 4. Our results are summarized in the Table below. Let VE denote that a voting equity contract can be used to implement the optimal solution to Problem 4, NVE denote that a non-voting equity contract can be used, and D denote that a debt contract can be used. Let None indicate that none of these three simple types of contracts can be used to implement the optimal allocation. The Table summarizes the sixteen possible constraint outcomes that arise from the two sets of constraints. The "appropriate" control structure clearly depends on which constraints hold (i.e., the primitives of the economy).

	(IICb) (IIRg)	**(IICb)**	**(IIRg)**	**Neither**
(ICb) (ICg)	VE, NVE, D	NVE, D	NVE	NVE
(ICg)	VE, D	D	None	None
(ICb)	VE	None	None	None
Neither	VE	None	None	None

The entries in the Table are derived as follows. Consider the first row, where (ICb) and (ICg) hold for the firm, so it reports truthfully even when auditing does not occur. When (IICb) and (IIRg) hold for investors, the Table indicates that all three types of contracts can be used to implement the optimal allocation. This occurs because it is optimal for both the firm and investors to follow the Pareto efficient plan τ^*.[30] Thus, when all four constraints hold the firm and investor interests are aligned so it does not matter which party is assigned control rights. Voting equity, non-voting equity, and debt can all be used. When the firm's constraints both hold without auditing but only (IICb) holds for investors, it does not matter which party is assigned control in the bad state because firm and investors'

[30]The argument is as follows. Suppose the firm can unilaterally decide when investment is terminated, and it chooses time t. Then investors receive D_t. Since both (ICb) and (ICg) hold for $A = \emptyset$, the firm reports truthfully and terminates investment the first time R_t switches to R_b. Now suppose investors can unilaterally decide when investment is terminated, and they choose time t. They receive D_t if R_t switches to R_b for the first time at t. Since (IICb) holds it is optimal for investors to stop at t if the state is bad. Further, since (IIRg) holds investors do not withdraw finance in good states.

interests are aligned. However, since (IIRg) does not hold, the firm must be given control in the good state since investors may wish to stop in some good states even though this is not Pareto efficient. Both non-voting equity and debt obviate this problem because they assign control to the firm in the good state. When the firm's constraints hold but only (IIRg) holds for investors, the firm must have control in the bad state but it does not matter who has control in the good states. Only non-voting equity achieves this.[31] If both firm constraints hold but neither investor constraint holds, only non-voting equity can be used because it gives the firm control in all states.

Consider the second row of the Table, where (ICg) holds for the firm but (ICb) does not for $(D_t, \emptyset)$, so auditing is necessary to ensure that the firm reports the bad state truthfully. In this case, if the firm has complete control (e.g., because of non-voting equity contracts) optimal investment plan τ^* cannot be implemented because a time period t and a stopping time τ exist such that continuing at time t using investment plan τ is better than stopping.[32] When the firm has control it will clearly take advantage of this opportunity. Investors, knowing this, will not sign such contracts at the outset. Assigning control rights to investors obviates this problem and allows optimal plan τ^* to be implemented. Whether debt or voting equity is optimal depends on the investors' constraints. When (IICb) and (IIRg) both hold either voting equity or debt can be used to implement the optimal allocation because investors follow τ^* in both the good and the bad state. In contrast, when only (IICb) holds, the firm must be given control in the good state so only debt is optimal.[33] When only (IIRg) holds or neither (IICb) or (IIRg) hold none of the simple contracts we consider can be used to implement the optimal allocation.

Finally, consider the last two rows of the Table. In case 3 only (ICb) holds for the firm when $(D_t, \emptyset)$ so the firm truthfully reports in the bad state

[31] Debt gives investors control in the bad state and voting equity gives investors control in all states, hence they cannot be used.

[32] The left-hand side of (ICb) is the payoff to the firm from stopping at time t. The right-hand side is the expected payoff from continuing. Since (ICb) is violated, the left-hand side is strictly less than the right-hand side for this stopping time τ.

[33] When (IIRg) does not hold, investors may stop in some good states. This could occur in an intertemporal contract if the firm paid investors a premium for entering the contract at the outset, but made lower future payments. Such contracts can be optimal if interest rates are time-dependent, and hence the problem is non-recursive. In such "front-end loaded" contracts, control must be assigned to the firm in good states.

without monitoring, but auditing is necessary to ensure that the firm reports the good state truthfully. Thus, control rights must be assigned to investors in good states. Only voting equity solves this problem. In all other cases none of the simple debt or equity contracts that we consider can be used to implement the optimal allocation. In case 4 neither incentive constraint holds for the firm, so clearly investors must have complete control when (IICb) and (IIRg) hold. In all other cases none of the simple contracts work.

The results summarized in the Table provide insight into factors which determine a firm's optimal capital structure. Should a firm raise capital by issuing debt, equity or some combination of the two? The well known Modigliani-Miller Theorem provides conditions under which the choice of debt versus equity is irrelevant. In contrast, the Table classifies situations under which the choice between the two instruments is irrelevant, situations where only equity finance is optimal, situations where only debt finance is optimal, and situations where neither type of simple finance can be used.[34] Most firms and corporate finance practitioners consider the firm's choice of capital structure to be a substantive problem. Thus, the Modigliani-Miller Theorem, even when "market imperfections" (e.g., taxes) are taken into account, is troublesome. Our results suggest that the basic corporate finance problem—how should a firm raise capital—is affected by at least two factors: the traditional cost of capital approach *and* implementation problems that arise from the economy's primitive structure that are "solved" by financial instruments with inherently different control properties.

7 Concluding Remarks

This paper contains two main results. First, we derive the structure of optimal intertemporal contracts under differential information and show that there are gains from intertemporal contracting that stem purely from information effects. Townsend (1982) and Green (1987) have shown that risk averse agents can use intertemporal contracts to attain allocations that are

[34]In most of the "None" cases, more sophisticated control structures can implement the optimal allocation. For example, debt contracts with a detailed set of covenants which assign control to investors in time periods where (ICb) does not hold and to the firm when (IICb) does not hold can implement the relevant case (3) and (4) contracts. Recall that these two constraints cannot both be violated simultaneously by (A2).

Pareto superior to those attainable with static contracts because such contracts provide agents with consumption insurance. In contrast, our gains from intertemporal contracting (with risk neutral agents) stem solely from information revelation that is possible in an intertemporal model but not a static one. These gains arise from the fact that the intertemporal structure of the model allows agents to reduce, and sometimes even eliminate, the ex post inefficiency inherent in the static costly state verification model.

Second, we analyze optimal contract implementation and control under differential information. Our results indicate that control problems arise in intertemporal contracting problems when agents have asymmetric continuation values that cannot be internalized. Specifically, Section 4 shows that asymmetric continuation values can sometimes be internalized via an appropriate reward structure. When the conditions for this to occur are satisfied, control problems do not arise. In contrast, Sections 5 and 6 show that when the firm has a propensity to cheat (because of limited liability) that cannot be costlessly internalized, auditing and control become important issues that are inherently related. The implementation analysis in Section 6 also provides insight into the role of alternative financial instruments (i.e., debt versus equity) in solving control problems in economies with differential information. The results suggest that our dynamic, stochastic contracting model with differential information may prove useful in uncovering the determinants of a firm's optimal capital structure. A more detailed analysis of this problem remains for future research.

Finally, in addition to the investment finance questions we address, our model may be of broader methodological interest. We analyze optimal incentive-constrained contracts in a dynamic, stochastic economy with differential information. History dependence is a well known and troublesome technical problem in such economies, and to make history dependence analytically tractable we introduce a mathematical tool known as "stopping times." History dependence arises in our model because investment termination decisions may depend on the entire history of Markov process realizations. Further, in the auditing model the time period in which audits occur is history dependent, thus economies with different histories have different information (and hence payoff) structures. Stopping times are useful because they permit the characterization of analytic solutions to both recursive *and* non-recursive (the more technically troublesome) economic problems.

Appendix A

We now derive technical results implied by the assumptions.

Lemma A. *Assume (A1), (A3) and (A4) hold. Then (A2) is fulfilled iff*

$$\delta p(b,g)R_g + \frac{\delta^2 p(b,g)p(g,b)R_b}{1-\delta p(g,g)R_g} + \delta p(b,b)R_b < 1. \tag{9}$$

Proof. We proceed as follows:[35] (i) Show that a violation of (9) implies a violation of (A2). (ii) Show that if (9) holds then (A2) must hold.

(i) If investors follow an alternative investment strategy but (9) does not hold (A2) is clearly violated since an investor would only adopt an alternative strategy if $E\left[\delta^\tau \prod_{k=1}^{\tau} R_k | R_0 = R_b\right] > 1$, a contradiction of (A2).

(ii) Assume (9) holds and let τ be an arbitrary stopping time such that $\tau < \infty$ a.e. (A3) implies that it is always optimal for investors to continue with the firm in the good state, so without loss of generality we can also assume this is the case for τ. Unless τ is already an investment strategy where no reinvestment is done in bad states, there can be countable many states R_b where investment with the firm is continued. Thus, there exists a sequence of stopping times τ_i, $i \in I\!N$, where

(a) $\lim_{i\to\infty} \tau_i = \tau$ and τ_1 is the investment strategy given by the left-hand side of (9), i.e., continue investment even though the Markov process realization is R_b until the next R_b realization. Hence

$$E\left[\delta^{\tau_1} \prod_{k=1}^{\tau_1} R_k \;\middle|\; R_0 = R_b\right] = \delta p(b,g)\tilde{R} + \delta p(b,b)R_b < 1. \tag{10}$$

[35] (9) is derived as follows. Suppose the realization at time t is bad but investors choose to adopt the following "alternative" investment strategy: continue investing until the next bad realization. The investors' expected payoff from this strategy is the left-hand side of (9) which can be written as: $p(b,g)\delta\tilde{R} + p(b,b)\delta R_b$. The first term is the probability the state changes from bad to good times the expected discounted payoff from continuing in the resulting good states until a bad state is realized. The second term is the probability the state remains bad times the payoff from investing for one additional bad period. $\tilde{R} = R_g + \sum_{i=1}^{\infty} \delta^i p(g,g)^{i-1} p(g,b) R_g^{i-1} R_b$, where R_g is the initial good realization, $p(g,g)^i p(g,b)$ is the probability of getting a string of (discounted) good realizations of length i and an $(i+1)$st realization which is R_b. By the formula for the geometric series, $\tilde{R} = R_g + \delta p(g,b)R_b/(1-\delta p(g,g)R_g)$.

(b) τ_i and τ_{i+1} differ only as follows: There exists exactly one state with a R_b realization where reinvestment takes place under τ_{i+1} but not under τ_i.[36]

(9) implies $E\left[\delta^{\tau_{i+1}} \prod_{k=1}^{\tau_{i+1}} R_k | R_0 = R_b\right] < E\left[\delta^{\tau_i} \prod_{k=1}^{\tau_i} R_k | R_0 = R_b\right]$, for every $i \in \mathbb{N}$. Note that $\lim_{i\to\infty} \delta^{\tau_i} \prod_{k=1}^{\tau_i} R_k = \delta^{\tau} \prod_{k=1}^{\tau} R_k$ a.e. since $\tau < \infty$ a.e. Fatou's Lemma therefore implies

$$E\left[\delta^{\tau} \prod_{k=1}^{\tau} R_k | R_0 = R_b\right] \leq \lim_{i\to\infty} E\left[\delta^{\tau_i} \prod_{k=1}^{\tau_i} R_k | R_0 = R_b\right]$$
$$< E\left[\delta^{\tau_1} \prod_{k=1}^{\tau_1} R_k | R_0 = R_b\right]. \tag{11}$$

(10) and (11) immediately imply that (A2) holds. This concludes the proof.

(A2) implies the payoff from investment is bounded if the state is R_b and Lemma A establishes this is true in bad states for all stopping times τ with $\tau < \infty$ a.e. We prove the payoff is also bounded in state R_g in the following Corollary (used in the proof of Theorem 3).

Corollary 1. *(A1)-(A4) imply that $E\left[\delta^{\tau} \prod_{k=1}^{\tau} R_k\right] < \infty$ for all stopping times τ with $\tau < \infty$ a.e.*

Proof. From Lemma 1 it follows that it is optimal for agents to continue investment as long as the state is R_g and to stop once the state switches to R_b. The payoff of such an investment strategy is $\sum_{k=1}^{\infty} \delta^t p(g,g)^{t-1} p(g,b) R_g^{t-1} R_b = \delta p(g,b) R_b / (1 - \delta p(g,g) R_g)$. This is bounded since $\delta p(g,g) \neq 1$, otherwise (9) cannot hold by Lemma A. This proves the Corollary.

The next Corollary is useful for computing the example in Section 4.

Corollary 2. *Assume that (A1)-(A4) hold. Let $\mathcal{H}$ be the set of all stopping times with $\tau \geq 1$. Then $\max_{\tau\in H} E\left[\delta^{\tau} \prod_{k=1}^{\tau} R_k \,\middle|\, R_0 = R_b\right] = \delta p(b,g)\tilde{R} + \delta p(b,b) R_b$, where $\tilde{R} = R_g + \delta p(g,b) R_b / (1 - \delta p(g,g) R_g)$.*

[36](b) implies that $[\delta p(b,b) R_b + \delta p(b,g) R_g + \delta^2 p(b,g) p(g,b) R_b / (1 - \delta p(g,g) R_g)]x < x$, where x denotes an agent's wealth, the left-hand side of the equation is the agent's expected return from following the "alternative" investment strategy, and the right-hand side is the agent's return from liquidating the risky investment when the first R_b is realized.

Proof. The result follows immediately from the proof of Lemma A: (11) proves τ^1 is the optimal stopping time. Thus, (10) proves the Corollary.

We now prove Claim 1 from Lemma 2 in Section 4

Proof of Claim 1. *The function* $\psi\colon [0,1) \to I\!R$ *defined by*

$$a \mapsto E\left[\delta^\tau \frac{\left(\prod_{k=1}^{\tau(\omega)} R_k(\omega) - a r_g^{\tau(\omega)}\right)^+}{1-a} \,\middle|\, R_0 = R_b\right] \tag{12}$$

is U-shaped and thus assumes only one local minimum.

Proof. Assume by way of contradiction there exist points $a_1 < \bar{a} < a_2$ such that $\psi(\bar{a}) > \psi(a_i)$, for $i = 1, 2$. (12) can be written as $a \mapsto \sum_{i=1}^{\infty} \lambda_i \frac{\left(l_i - a r_g^{\tau_i}\right)^+}{1-a} + \sum_{i=1}^{\infty} \mu_i \frac{\left(m_i - a r_g^{\tau_i}\right)^+}{1-a}$, where $l_i \leq r_g^{\tau_i}$ and $m_i > r_g^{\tau_i}$.[37] Without loss of generality assume that $l_i < l_{i+1}$ for every $i = 1, \ldots, m$. Let $\psi_n\colon [a_1, a_2] \to I\!R$ be defined by $a \mapsto \sum_{i=1}^{n} \lambda_i \frac{\left(l_i - a r_g^{\tau_i}\right)^+}{1-a} + \sum_{i=1}^{n} \mu_i \frac{\left(m_i - a r_g^{\tau_i}\right)^+}{1-a}$. Then $\lim_{n \to \infty} \psi_n(a) = \psi(a)$ for every a. To derive a contradiction, it is sufficient to prove that every ψ_n is U-shaped on $[a_1, a_2]$. The derivative of ψ_n is given by[38]

$$a \mapsto \frac{1}{(1-a)^2} \left(\sum_{i=k}^{n} \lambda_i (l_i - r_g^{\tau_i}) + \sum_{i=1}^{n} \mu_i (m_i - r_g^{\tau_i}) \right),$$

where k is the first index for which $l_i - a r_g^{\tau_i} \geq 0$, the first sum is always negative, and the second sum is always positive. As a increases k increases, thus the derivative can only switch once from a negative to a positive sign as a increases. This implies that ψ_n can have only one local minimum. Thus, ψ_n is U-shaped on $[0, 1)$, i.e. $\psi_n(\bar{a}) \leq \psi_n(a_i)$, $i = 1, 2$. Take the limit for $n \to \infty$ to get $\psi(\bar{a}) \leq \psi(a_i)$, $i = 1, 2$. This contradiction proves the claim.

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[37]This follows since the Markov process has a discrete state space. l_i and τ_i denote one of finitely many possible realizations of $\prod_{k=1}^{\tau} R_k$ and τ, respectively, and λ_i is the probability that such a realization occurs. The interpretation of m_i and μ_i is similar.

[38]Obviously, ψ is not differentiable at points a where $l_i = a r_g^{\tau_i}$ for some i. However, it is sufficient that the left-hand and right-hand derivatives exist.

Appendix B

In this Appendix we give a revelation principle argument to show that the restriction that the firm reports truthfully in all problems is without loss of generality. In addition, we prove Lemma 3. We begin with some definitions.

Contract set $\mathcal{C}$ is a set of tuples $(\tau^1, \tau^2, (r_k)_{k\in \mathbb{N}}, (h_k)_{k\in \mathbb{N}}, (\varsigma_k)_{k\in \mathbb{N}}, \phi)$, with the following properties:

(i) $h_k\colon \Omega \to \mathcal{S}$, $k \in \mathbb{N}$ are report functions measurable with respect to $\mathcal{F}_k$.
(ii) τ^1 is a stopping time with respect to agents' information $\tilde{\mathcal{F}}_k$, $k \in \mathbb{N}$.
(iii) τ^2 is a stopping time with respect to the firm's information $\mathcal{F}_k$, $k \in \mathbb{N}$.
(iv) $r_k\colon \Omega \to \mathbb{R}$ is a payment function measurable with respect to $\tilde{\mathcal{F}}_k$;
(v) for every j, ς_j is a audit stopping time with respect to $\tilde{\mathcal{F}}_k$, $k \in \mathbb{N}$.
(vi) $\phi\colon \mathbb{R} \to \mathbb{R}$ is a penalty function with $0 \le \phi(x) \le x$, for every $x \in \mathbb{R}$.

Contracts $C \in \mathcal{C}$ satisfy (i)–(v) and:

(vi) $\tau^2(\omega) \ge \tau^1(\omega)$ for a.e. ω;
(vii) $\varsigma^i(\omega) \le \varsigma^{i+1}(\omega)$ for a.e. ω, and for every $i \in \mathbb{N}$.

The investors' information set, $\tilde{\mathcal{F}}_k$, clearly depends on h_k, $k \in \mathbb{N}$ and on ς^i, $i \in \mathbb{N}$. However, h_k and ς^i in turn are defined with respect to this information. We solve this problem by defining the information set inductively.[39]

Consider the Stackelberg game between the firm and investors:

(a) Investors agree to a contract C if they receive at least their reservation utility. Let $\mathcal{R} \subset \mathcal{C}$ be the set of all contracts that investors agree to.
(b) The firm chooses a contract in $\mathcal{R}$ which maximizes its expected profit.

We now specify $\mathcal{R}$: Denote by $E\Pi(C)$ the firm's expected profit, let $Eu(C)$ denote the investors' expected utility under contract C, and let $h\colon \Omega \to \Omega$ be defined by $h(\omega) = (h_1(\omega), h_2(\omega), \ldots) = \prod_{k=1}^{\infty} h_k(\omega)$. Thus, h maps any state ω into the reported state $h(\omega)$. Then $\mathcal{R}$ is the set of contracts $\bar{C} = (\bar{\tau}^1, \bar{\tau}^2, (\bar{r}_k)_{k\in \mathbb{N}}, (\bar{h}_k)_{k\in \mathbb{N}}, (\bar{\varsigma}_k)_{k\in \mathbb{N}}, \phi) \in \mathcal{C}$ such that:

(1) $\bar{C} \in \arg\max_{C\in\mathcal{C}} E\Pi(C)$, subject to:
 (i) $\tau^1(\omega) = \bar{\tau}^1(\bar{h}^{-1}(h(\omega)))$, a.e.;
 (ii) $r_k(\omega) = \bar{r}_k(\bar{h}^{-1}(h(\omega)))$, a.e. for every k;

[39] Let $\tilde{\mathcal{F}}_k^0 = \varsigma(h_1, \ldots, h_k)$, i.e., the σ-algebra generated by announcements in the first k periods. Assume we have constructed the information sets generated by h_j, $j \le k$ and by ς^i, $i \le m$. Denote these information sets by $\tilde{\mathcal{F}}_k^m$. Define $L_i = \{\omega\colon \varsigma^{m+1}(\omega) = i\}$. Then let $\mathcal{G}_i$ be the σ-algebra given by $\{A\colon A \cap L_i \in \tilde{\mathcal{F}}_i \text{ and } A \setminus L_i \in \tilde{\mathcal{F}}_k^m\}$. Thus, $\tilde{\mathcal{F}}_k^{m+1} = \bigvee_{i=1}^{k} \mathcal{G}_i$ is the investors' information generated by h_j, $j \le m+1$ and $\tilde{\mathcal{F}}_k = \bigvee_{i=1}^{\infty} \tilde{\mathcal{F}}_k^i$ for every $k \in \mathbb{N}$ is the investors' information set.

(iii) $\varsigma^k(\omega) = \bar{\varsigma}^k(\bar{h}^{-1}(h(\omega)))$, a.e. for every k;
(iv) $\phi(\omega) = \bar{\phi}(\omega))$ a.e.;
(2) $Eu(\bar{C}) \geq 1$.

The investors' reaction function is completely described by $\mathcal{R}$: They agree to contract C if $C \in \mathcal{R}$, and reject it otherwise. (1) specifies that the firm's report function $\bar{h}$ and stopping time $\bar{\tau}^2$ are optimal among all possible report plans h, and all possible stopping times τ^2. Note that $\bar{r}_k$, $\bar{\varsigma}^k$, and $\bar{\tau}^1$ depend on the investors' information generated by reports h. Thus, these elements of the contract are implicitly changed when the report function is changed from $\bar{h}$ to h. The way in which they are changed (to r_k, ς^k, and τ^1) is formally described by constraints (i), (ii), and (iii).[40] Constraint (iv) says that ϕ is an aspect of the contract on which the firm cannot renege, i.e., there is an ex-post enforcement mechanism which preserves the contract. The firm, as the Stackelberg leader therefore solves:

$$\max_{C \in \mathcal{R}} E\Pi(C). \tag{13}$$

We now we derive the profit function, compute the investors' utility, and give a revelation principle argument to show that only contracts which involve full revelation of information need be considered. Problem (13) then translates into the problem of finding an optimal contract subject to an incentive constraint and an individual rationality constraint.

To derive the profit function, note that firm misreports can be detected in two ways:

(a) At time k, the firm makes an announcement which triggers withdrawal. i.e., $\tau^1(\omega) = k$, but $\prod_{j=1}^{\tau^1(\omega)} R_j < \prod_{j=1}^{\tau^1(\omega)} r_j(h(\omega))$, (the firm's liabilities exceed its assets).

(b) At time k, the firm makes an announcement which triggers verification. i.e., $\varsigma^j(\omega) = k$ and $\tau^1(\omega) \geq k$,[41] and there was a current or past misreport,

[40] For example, consider the following situation which describes a simplified version of the model. There are three states in Ω denoted by a, b, and c. $\bar{h}$ is given by $\bar{h}(a) = a$, $\bar{h}(b) = c$, and $\bar{h}(c) = b$. Clearly, $\bar{h}$ generates a σ-algebra, corresponding to full information. Thus, payment function $\bar{r}$ defined by $\bar{r}(a) = 1$, $\bar{r}(b) = 2$, $\bar{r}(c) = 3$ is measurable with respect to the investors' information. Assume the firm switches to report function h defined by $h(a) = h(b) = b$, $h(c) = a$. Then $r(a) = \bar{r}(\bar{h}^{-1}h(a)) = \bar{r}(c)$. Similarly, it follows that $r(b) = \bar{r}(\bar{h}^{-1}h(b)) = \bar{r}(c)$, and $r(c) = \bar{r}(\bar{h}^{-1}h(c)) = \bar{r}(a)$. Thus a payment function r which fulfills the constraint $r(\omega) = \bar{r}(\bar{h}^{-1}(h(\omega)))$ is given by $r(a) = r(b) = \bar{r}(c) = 3$, and $r(c) = \bar{r}(a) = 1$, where $\bar{r}$ is measurable with respect to the information generated by h.

[41] This means that investors have not already withdrawn their investment.

so there exists a $j \leq k$ such that $h_j(\omega) \neq R_j(\omega)$.

Misreports are not detected otherwise. We now describe the set of states in which these two types of misreports can occur. Denote this set by $A = A_1 \cup A_2$, where A_1 and A_2 correspond to (a) and (b) above. Thus,

$$A_1 = \left\{ \omega : \prod_{j=1}^{\tau^1(\omega)} R_j(\omega) < \prod_{j=1}^{\tau^1(\omega)} r_j(h(\omega)) \right\}$$

$$A_2^i = \left\{ \omega : \varsigma^i(\omega) = k \text{ and } \exists j \leq k,\ h_j(\omega) \neq R_j(\omega) \text{ and } \tau^1(\omega) \geq k \right\}$$

$$A_2 = \bigcup_{i=1}^{\infty} A_2^i$$

For all $\omega \in A$, investors can impose penalty ϕ on the firm, instead of being paid according to the payment schedule r. Define $B_0 = A_1 \setminus A_2$, $B_1 = A_2^1$, $B_2 = A_2^2 \setminus A_2^1$, and in general $B_i = A_1^i \setminus \bigcup_{j=1}^{i} B_i$. Then the firm's expected profit is given by

$$E(\Pi(C)) = \int_{\Omega \setminus A} \delta^{\tau^2(\omega)} \left(\prod_{k=1}^{\tau^1(\omega)} R_k(\omega) - \prod_{k=1}^{\tau^1} r_k^h(\omega)) \right) \prod_{k=\tau^1(\omega)+1}^{\tau^2(\omega)} R_k(\omega)\, dP(\omega)$$
$$+ \int_{B_0} \delta^{\tau^1(\omega)} \left(\prod_{k=1}^{\tau^1(\omega)} R_k(\omega) - \phi \left(\prod_{k=1}^{\tau^1(\omega)} R_k(\omega) \right) \right) dP(\omega)$$
$$+ \sum_{i=1}^{\infty} \int_{B_i} \delta^{\varsigma^i(\omega)} \left(\prod_{k=1}^{\varsigma^i(\omega)} R_k(\omega) - \phi \left(\prod_{k=1}^{\varsigma^i(\omega)} R_k(\omega) \right) \right) dP(\omega).$$

To compute the investors' utility, note that in the derivation of the firm's expected profit we assumed that the firm is shut down when a misreport is detected. In such states, investors impose penalty ϕ. On B_0, the firm reports it is not able to pay, but there was no previous auditing (A_2 is subtracted from B_0). Hence the penalty is imposed, and investment is stopped at time $\tau^1(\omega)$. On sets B_i, $i > 0$, misreports are detected by auditing since B_i contains all states ω such that a misreport is detected the ith time auditing occurs but not previously. Thus, investment stops at time $\varsigma^i(\omega)$ and penalty ϕ is imposed. This penalty is a transfer from the firm to investors; no deadweight loss is involved. Thus, an investor's payoff is given by

$$E(u(C)) = \int_{\Omega \setminus A} \delta^{\tau^1(\omega)} \prod_{k=1}^{\tau^1(\omega)} r_k(\omega))\, dP(\omega) + \int_{B_0} \delta^{\tau^1(\omega)} \phi \left(\prod_{k=1}^{\tau^1(\omega)} R_k(\omega) \right) dP(\omega)$$

$$+\sum_{i=1}^{\infty}\int_{B_i}\delta^{\varsigma^i(\omega)}\phi\left(\prod_{k=1}^{\varsigma^i(\omega)}R_k(\omega)\right)dP(\omega)-\sum_{i=1}^{\infty}\int_{\{\omega:\exists k,\ \varsigma^k(\omega)=i\}}\delta^i c\,dP(\omega).$$

We now give the revelation principle argument. Assume that $\bar{C}$ is a solution to Problem (13). Then $E\Pi(\bar{C}) \geq E\Pi(C)$ for all $C \in \mathcal{R}$. We proceed as follows: We first show that we can replace *any* contract C by a contract $\hat{C}$ for which $h(\omega) = \omega$, which we call a "truth-telling contract" (thus, $\bar{C}$ can be replaced by a truth-telling contract). We then show that under this transformation Problem (13) becomes Problem 3, where we maximize over the set of all truth-telling contracts, subject to an individual rationality constraint and an incentive compatibility constraint.

Let $C \in \mathcal{C}$. The corresponding truth-telling contract is denoted by $(\hat{\tau}^1, \hat{\tau}^2, (\hat{r}_k)_{k\in\boldsymbol{N}}, (\hat{\varsigma}_k)_{k\in\boldsymbol{N}}, \hat{\phi})$. Derive the new contract by redefining the original contract on each of the sets B_i. We need not redefine the contract on the complement of A. All components of the contract which we do not explicitly redefine remain unchanged. Start on B_0. Let $W_k = \{\omega : \tau^1(\omega) = i\}$. Define r_i on W_k as follows:

$$\hat{r}_k(\omega) = \frac{\phi\left(\prod_{i=1}^{k} R_i(\omega)\right)}{\prod_{i=1}^{k-1}\hat{r}_i(\omega)} \tag{14}$$

Clearly, the resulting $\hat{r}_k$ is $\mathcal{F}_k$-measurable on B_0. All other components of the contract remain unchanged on B_0. Next redefine the contract on each of the sets B_i, $i \geq 1$. Redefine r_k as in (14). Further, set $\tau^1(\omega) = \varsigma^i(\omega)$ on B_i. All other components of the contract remain unchanged on B_i, $i \geq 1$.

For the redefined contract we therefore get

$$E(\Pi(C)) = \int_{\Omega}\delta^{\hat{\tau}^2(\omega)}\left(\prod_{k=1}^{\hat{\tau}^1(\omega)}R_k(\omega) - \prod_{k=1}^{\hat{\tau}^1}\hat{r}_k^h(\omega))\right)\prod_{k=\hat{\tau}^1(\omega)+1}^{\hat{\tau}^2(\omega)}R_k(\omega)\,dP(\omega) \tag{15}$$

$$E(u(C)) = \int_{\Omega}\delta^{\hat{\tau}^1(\omega)}\prod_{k=1}^{\hat{\tau}^1(\omega)}\hat{r}_k(\omega))\,dP(\omega). \tag{16}$$

The constraint in (13) and (16) imply the (IR) constraint $E\left[\delta^{\hat{\tau}^1}\prod_{k=1}^{\hat{\tau}^1}\hat{r}_k\right] - E\left[\sum_{k=1}^{\tau^1}\delta^k c n_k\right] \geq 1$, where $n_k(\omega) = \left\{\omega : \exists k,\ \varsigma^k(\omega) = i\right\}$.

Since (16) is (IR) in Problem 3 and (15) is the objective, it remains to derive (ICC). Define $\tau^1(\omega) = \hat{\tau}^1(h(\omega))$, let $\varsigma^i(\omega) = \hat{\varsigma}^i(h(\omega))$ and $r_k(\omega) =$

$\hat{r}_k(h(\omega))$. Then $\tau^1(\omega) = \hat{\tau}^1(\bar{h}^{-1}(h_1(\omega)))$, $\varsigma^k(\omega) = \hat{\varsigma}^k(\bar{h}^{-1}(h_1(\omega)))$, and $r_k(\omega) = r_k(\bar{h}^{-1}(h_1(\omega)))$, where $h_1(\omega) = \bar{h}(h(\omega))$. Denote this new contract by C. Conditions (i)–(iv) of (1) in definition of $\mathcal{R}$ are satisfied, and we get $EU(\hat{C}) \geq EU(C)$. This means the incentive constraint is

$$\int_\Omega \delta^{\hat{\tau}^2(\omega)} \left(\prod_{k=1}^{\hat{\tau}^1(\omega)} R_k(\omega) - \prod_{k=1}^{\hat{\tau}^1} \hat{r}_k^h(\omega)) \right) \prod_{k=\hat{\tau}^1(\omega)+1}^{\hat{\tau}^2(\omega)} R_k(\omega)\, dP(\omega)$$
$$\geq \int_{\Omega\setminus A} \delta^{\hat{\tau}^2(h(\omega))} \left(\prod_{k=1}^{\hat{\tau}^1(h(\omega))} R_k(\omega) - \prod_{k=1}^{\hat{\tau}^1(h(\omega))} \hat{r}_k(h(\omega)) \right) \prod_{k=\hat{\tau}^1(h(\omega))+1}^{\hat{\tau}^2(h(\omega))} R_k(\omega)\, dP(\omega)$$
$$+ \int_{B_0} \delta^{\hat{\tau}^1(h(\omega))} \left(\prod_{k=1}^{\hat{\tau}^1(h(\omega))} R_k(\omega) - \phi\left(\prod_{k=1}^{\hat{\tau}^1(h(\omega))} R_k(\omega) \right) \right) dP(\omega)$$
$$+ \sum_{i=1}^{\infty} \int_{B_i} \delta^{\hat{\varsigma}^i(h(\omega))} \left(\prod_{k=1}^{\hat{\varsigma}^i(h(\omega))} R_k(\omega) - \phi\left(\prod_{k=1}^{\hat{\varsigma}^i(h(\omega))} R_k(\omega) \right) \right) dP(\omega),$$

for every report function. This concludes the revelation principle argument.

Assume without loss of generality that ϕ is as large as possible. In fact, any solution to the problem of finding an optimal contract $\bar{C}$, subject to (IR) and (ICC) can be supported by a contract with $\phi(x) = x$ instead of the arbitrary $\bar{\phi}$ of contract $\bar{C}$. Thus, the incentive constraint becomes

$$\int_\Omega \delta^{\hat{\tau}^2(\omega)} \left(\prod_{k=1}^{\hat{\tau}^1(\omega)} R_k(\omega) - \prod_{k=1}^{\hat{\tau}^1} \hat{r}_k^h(\omega)) \right) \prod_{k=\hat{\tau}^1(\omega)+1}^{\hat{\tau}^2(\omega)} R_k(\omega)\, dP(\omega)$$
$$\geq \int_{\Omega\setminus A} \delta^{\hat{\tau}^2(h(\omega))} \left(\prod_{k=1}^{\hat{\tau}^1(h(\omega))} R_k(\omega) - \prod_{k=1}^{\hat{\tau}^1(h(\omega))} \hat{r}_k(h(\omega)) \right) \prod_{k=\hat{\tau}^1(h(\omega))+1}^{\hat{\tau}^2(h(\omega))} R_k(\omega)\, dP(\omega).$$

for every h, since all other terms are zero. By Lemma 3 (proved below), we can restrict our analysis to reporting plans $\mathcal{H}(\varsigma^i, \tau^1)$. Nevertheless, it can occur that $\tau^1(h(\omega)) = t$ and the firm's liabilities exceed its asset at t. In such a case the firm's payoff is zero since the penalty is maximal. This corresponds to rewriting the right-hand side of the above inequality as in (ICC) in Problem 3.

Finally, we prove Lemma 3.

Proof of Lemma 3. Consider an arbitrary report strategy h such that $\tau^1(h(\omega)) < \infty$ a.e. To prove the Lemma, it is sufficient to construct a report strategy $\bar{h}$ which is not detectable via auditing, and for which the firm's profit is higher than under h. We start by defining an event tree: Define the following equivalence relation on $\Omega \times \mathbb{N}$. Let $(\omega, t) \sim (\omega', t')$ if and only if $t = t'$ and $R_k(\omega) = R_k(\omega')$ for every $k \leq t$. Let $\mathcal{E}$ be the set of all equivalence classes on $\Omega \times \mathbb{N}$ with respect to "$\sim$." Then $\mathcal{E}$ is the event tree. Introduce the natural ordering on $\mathcal{E}$: Let $s_1 = (\omega_1, t_1)$ then $s_1 \preceq s_2$ if and only if $t_1 \leq t_2$ and $R_k(\omega_1) = R_k(\omega_2)$ for every $k \leq t_1$. Think of h as defined on $\mathcal{E}$. Let $\mathcal{M}$ denote the set of all nodes in the event tree such that (a) for every $s \in \mathcal{M}$ the state is misreported; (b) for $s \in \mathcal{M}$ no misreport is made at every $s' \prec s$. Without loss of generality assume that for every $s \in \mathcal{M}$ there is a way to report the future nodes such that auditing never occurs and agents withdraw after finitely many periods. Otherwise, it is (weakly) optimal to report s truthfully since the firm's payoff is zero if a misreport is detected via auditing.

Assume that for an $s = (t, \omega) \in \mathcal{M}$, and for some future path of states $s_{t+1}, \ldots, s_{t+m}$, the functions h_k, $k = t, \ldots, t+m$ specify reports which trigger an audit at s_{t+m}.[42] Then choose the largest k with $t + m \geq k \geq t$ such that given the reports in states s_{t+1}, s_{t+k-1} it is possible to report future states such that an audit does not occur before investors withdraw their investment.[43] By redefining h on s_{t+k+i}, $i \geq 0$, we can therefore avoid a misreport which is detected via auditing. The firm's payoff must weakly increase since its profit is zero whenever a misreport is detected. Using this construction, we can derive a sequence of report functions h_n, $n \in \mathbb{N}$ such that h_n converges to report function $\bar{h}$ as $(n \to \infty)$ for which misreports are never detected by auditing. By construction the following property holds: For every $\omega \in \Omega$ there exists N such that $h_n(\omega) = \bar{h}(\omega)$ for every $n \geq N$. As a consequence all terms in Problem 3 converge when we substitute h_n for h and take the limit for $n \to \infty$. Since the firm's expected profit is at least as high under h_n as under h_{n-1}, the firm's expected profit under $\bar{h}$ is at least as high as under h. Since h is arbitrary, and since no misreports are detected by auditing under $\bar{h}$, this concludes the proof.

[42]Obviously, assume that $\tau^1 \geq t + m$ i.e., agents have not stopped investing.

[43]Such a k must exist since it is possible for every $s \in \mathcal{M}$ to make reports which do not trigger an audit before investors withdraw.

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